Marlbrook Primary School
Teaching School Alliance

Affiliated with Little Dewchurch C of E and Ewyas Harold Primary Schools

Help your child in Maths

- Understand your child's mental strategies and written methods
- Encourage your child to know and understand multiplication facts
- Ideas for using maths at home

The maths work your child is doing at school may look very different to the kind of 'sums' you remember. This is because children are encouraged to work mentally, where possible, using personal jottings to help support their thinking. Even when children are taught more formal written methods (from year 3 onwards), they are only encouraged to use these methods for calculations they cannot solve in their heads.

Discussing the efficiency and suitability of different strategies is an important part of maths lessons.

Encourage your child to explain their thinking.
Talk to your child about how <u>you</u> work things out.

When faced with a calculation problem, encourage your child to ask…

- Can I do this in my head?
- Could I do this in my head using drawings or jottings to help me?
- Do I need to use a written method?

Also help your child to estimate and then check the answer. Encourage them to ask…

Is the answer sensible?

How could I check?

MENTAL CALCULATION STRATEGIES

We are all different and think in different ways. There are lots of different ways of calculating 'in your head'. It's important that we recognise the strategies that children use so that we can support their learning.

Mental Strategies for adding and subtracting

Counting

- Counting on from first number
  (4 + 5): *'4.. . 5, 6, 7, 8, 9 . . . it's* nine'
- Counting on from larger
  (5 + 6): *'It's 11. I did it in my head. . . I took the big number first and added 5* in my *head'*
- Counting back from
  (7-3) : *'Four. . . I counted backwards. . . 7. . . 6,5, 4'*
- Counting back to
  (7 - 3): *'7.. . 6, 5, 4, 3. It's four'*
- Counting up from (complementary addition)
  (13 - 6):'Seven . . . *I counted up on my fingers'*

Using Known Facts

- Doubles fact (subtraction)
  (18 - 9) : *'Nine. . . 'cos I know that nine and* nine *is 18'*
- Near-doubles (addition)
  (8 + 5): *'13. . . because 8* and *8 is 16. . . take away 3'*
- Near-doubles (subtraction)
  (9 - 5): 'Four. . . because 10 take 5 is 5. . . and 9 is one down from *10'*
- Subtraction as the inverse of addition
  (7 - 3): *'Four. . . I knew 4 and 3 was 7. . . and I just took away 3'*

- Using fives
  (6 + 7): *'13.. . I took 5 out of the 6 and 5 out of the 7 and I was left with 3'*
- Bridging through ten (addition)
  (8 + 6): *'If 8 is two less than 10. . . add two off the 6. . . then. . . all the leftovers from before . . . so you just put them to 14'*
- Bridging through ten (subtraction)
  (12 - 4): *'Eight. . . I knew that if you take away two. . . that's 10. . . and you've got another two left, and you take away that and it's 8*
- Compensation
  (9 + 5): *'14.. . ten and five is 15 . . . and so 9 and 5 would be 14'*

There are lots of different methods…so if my child is struggling, which ones are the best to use?

The most common methods, and those that are more useful later on:

- bridging through ten (up and down);
- partitioning single digit numbers (this has huge implications for other operations too);
- compensation (for adding or subtracting 9).

As these strategies are most useful in later areas of maths, if we ever need to 'teach' a strategy, these are the ones that we should 'teach'.

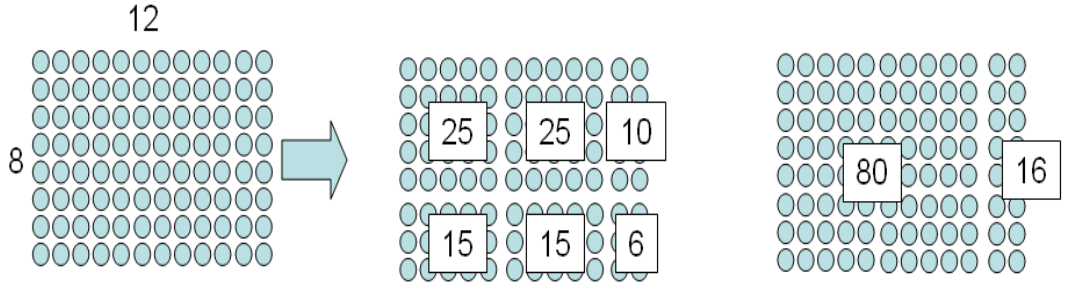
Encourage your child to show you how they have worked it out!

Mental strategies for multiplying and dividing

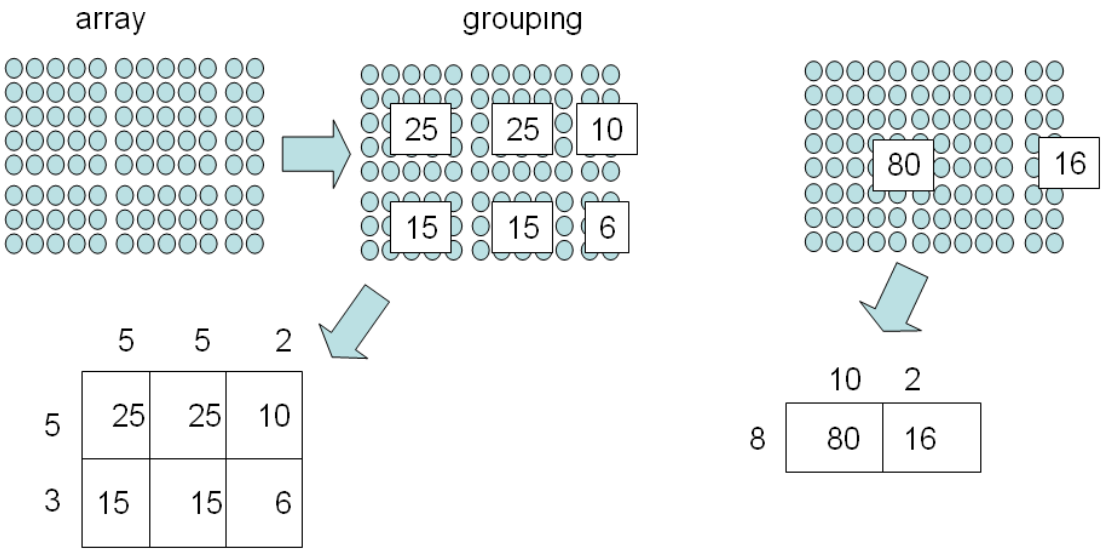
While considering multiplication as continuous addition often comes easy to early learners, children need to understand multiplication as a binary operation as this will help with later development. Therefore, while replication can help with understanding written methods, it is not as useful as other methods in developing mental strategies.

Below are two representations of 3 x 8 = 24. While these are common images to use in explaining multiplication (and division) for children who are learning the operation or trying to visualise a word problem, it is not particularly useful in covering all of the aspects of multiplication or establishing the links between this operation and other aspects of numeracy.

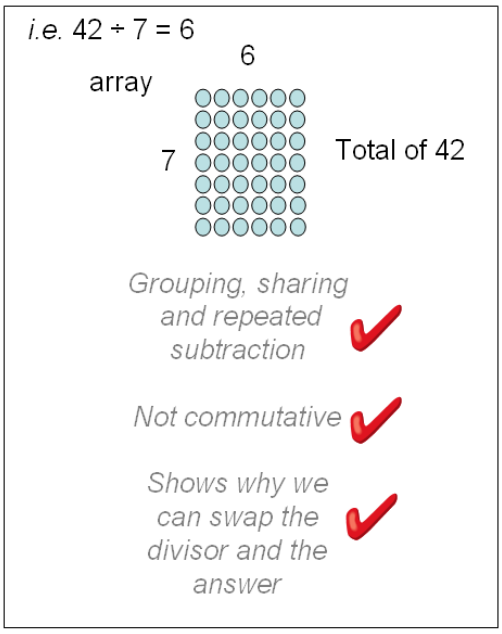
The best way to demonstrate multiplication and division, and a common way for children to imagine what multiplication and division actually are, is to imagine an array. Using arrays demonstrates a very important point; it doesn't matter whether you multiply 12 by 8, or 8 by 12; the answer is the same (multiplication is *commutative*).

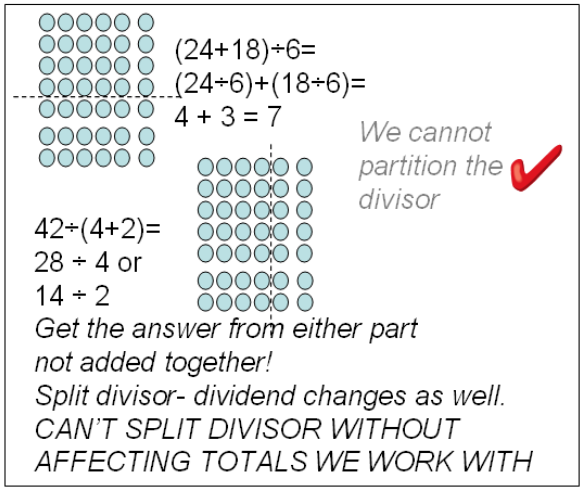


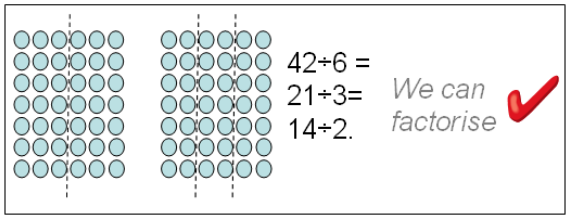
In addition, arrays are easily broken down into more manageable units. This encourages children to use known, simpler, number facts for calculations, helps with the understanding of calculations associated with shape and measure, and leads directly on to written methods.



Arrays are also good ways to demonstrate and imagine division.

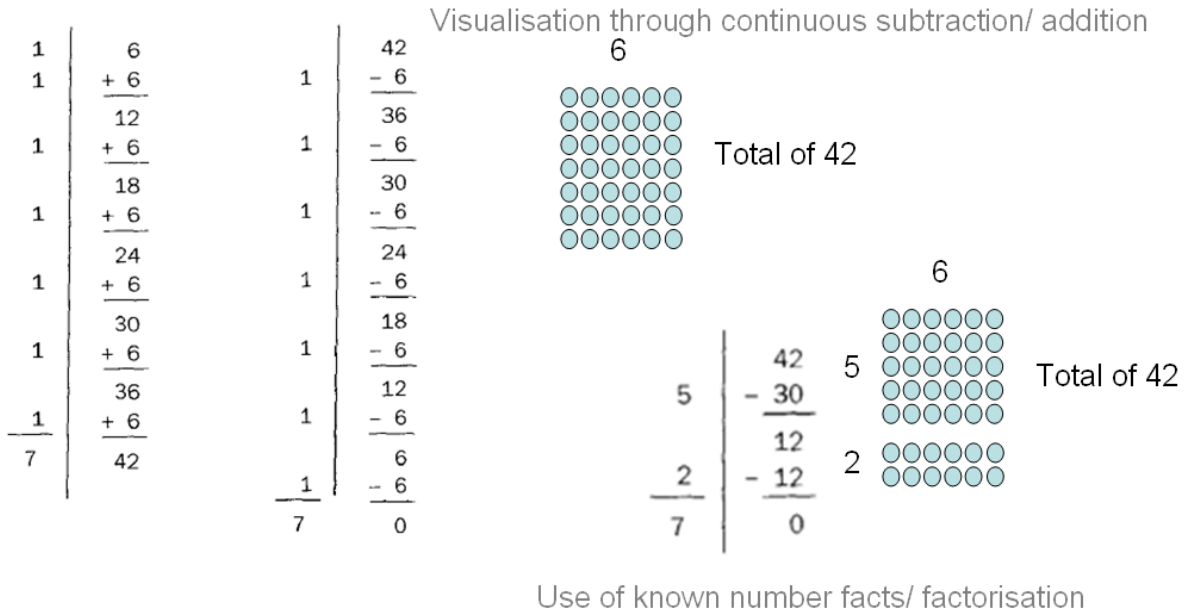






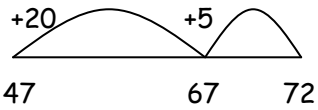
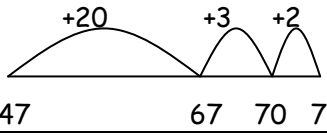
The diagrams show how arrays allow children to recognise all of the different aspects of division (that are often a bit of a mystery). Arrays allow us to illustrate grouping and sharing, demonstrate that the operation is not commutative and also how we can swap the divisor and the answer. They also allow children to understand some other aspects that year 5 and 6 children find difficult to learn; particularly fractions and how they can be 'simplified' by using factors.

Many children say that the written method for division- 'chunking'- is one of their least favourite activities! This is often because they do not have the experiences or images to draw on, as with fractions and related areas. Using arrays helps those that have mental images based on continual addition and subtraction relate to the simplest form of 'chunking' where the divisor is used rather than drawing on known number facts. When the understanding of the method and the array is on a sound footing, it then becomes easy to relate to chunking using factorisation/number facts.

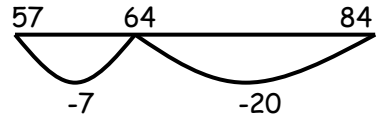
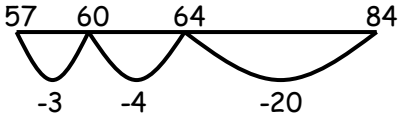
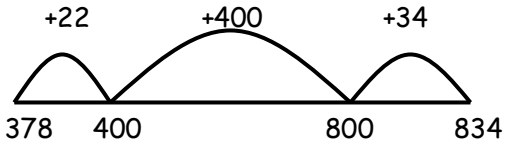


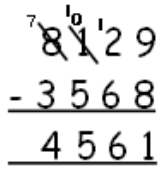
Therefore, whenever you talk about multiplication or division with your child, drawing the problem as an array is the best way of helping their understanding.

WRITTEN METHODS FOR ADDITION

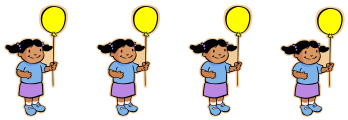
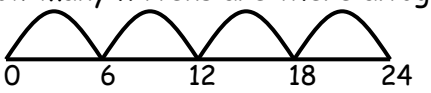
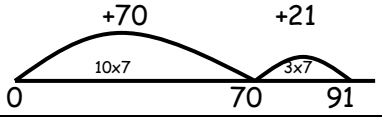
2+3= At a party, I eat 2 cakes and my friend eats 3. How many cakes did we eat altogether? 	Children could draw a picture to help them work out the answer.
7+4= 7 people are on the bus. 4 more get on at the next stop. How many people are on the bus now? ☺☺☺☺☺☺☺ ☺☺☺☺	Children could use dots or tally marks to represent objects (quicker than drawing a picture)
47+25= My sunflower is 47cm tall. It grows another 25cm. How tall is it now?  or  	Drawing an empty number line helps children to record the steps they have taken in a calculation (start on 47, +20, then +5). This is much more efficient than counting on in ones.
487+546= There are 487 boys and 546 girls in a school. How many children are there altogether? 5 4 6 + 4 8 7 1 3 1 2 0 9 0 0 1 0 3 3	Children will be taught written methods for those calculations they cannot do 'in their heads'. Expanded methods build on mental methods and make the value of the digits clear to children. The language used is very important (6+7, 40+80, 500+400, then 900+120+13 - *add this mentally NOT in columns*).
12 786 + 2 568= 12 786 people visited the museum last year. The numbers increased by 2 568 this year. How many people altogether visited this year? 1 2 7 8 6 + 2 5 6 8 1 5 3 5 4 1 1 1	When children are confident using the expanded method, this can be 'squashed' into the traditional compact method. ***This is nationally expected by the end of year 4.***

WRITTEN METHODS FOR SUBTRACTION

5-2= I had five balloons. Two burst. How many did I have left?  A teddy bear costs £5 and a doll costs £2. How much more does the bear cost? 	Drawing a picture helps children to visualise the problem.
7-3= Mum baked 7 biscuits. I ate 3. How many were left? 𝍸 |||| Take away Lisa has 7 felt tip pens and Tim has 3. How many more does Lisa have? ●●●●●●● ●●● Find the difference	Using dots or tally marks is quicker than drawing a detailed picture.
84-27= I cut 27cm off a ribbon measuring 84cm. How much is left?  or  	Children could count back using an empty number line. This is a really good way for them to record the steps they have taken (start on 84, -20, then -7).
834-378= The library owns 834 books. 378 are out on loan. How many are on the shelves?  2 2 (400) 4 0 0 (800) 3 4 (834) 4 5 7	Children could count up (from the smallest number to the biggest) using an empty number line. It is easiest to count up to a multiple of 10 or 100 (a friendly number). The steps can also be recorded vertically. This method works really well with any numbers, including decimals!

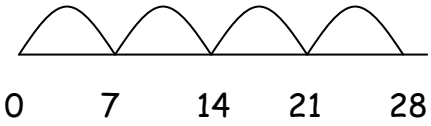
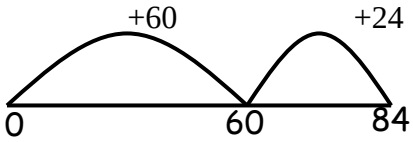
8 129 - 3 568= 8 129 people visited the museum last year. The numbers decreased by 3 568 this year. How many people altogether visited this year?  $\begin{array}{r} {}^{7}\not{8}\,{}^{1}\not{1}\,{}^{10}2\,9 \\ -\,3\,5\,6\,8 \\ \hline 4\,5\,6\,1 \end{array}$	When children are confident and understand the process of subtraction, they will then start to develop their understanding of the traditional decomposition method. ***This is nationally expected by the end of year 4.***

WRITTEN METHODS FOR MULTIPLICATION

2x4= Each child has two eyes. How many eyes do four children have?  2 + 2 + 2 + 2	Again a picture can be useful.
5x3= There are 5 cakes in a pack. How many cakes in 3 packs? ●●●●● ●●●●● ●●●●● 5 + 5 + 5	Dots or tally marks are often drawn in groups. This shows 3 groups of 5.
4x3= A chew costs 4p. How much do 3 chews cost? ●●●● ●●● ●●●● ●●● ●●●● or ●●● ●●●	Drawing an array (3 rows of 4 or 3 columns of 4) gives children an image of the answer. It also helps develop the understanding that 4x3 is the same as 3x4.
6x4= There are 4 cats. Each cat has 6 kittens. How many kittens are there altogether?  0 6 12 18 24	Children could count on in equal steps, recording each jump on an empty number line. This shows 4 jumps of 6.
13x7= There are 13 biscuits in a packet. How many biscuits in 7 packets?  	When numbers get bigger, it is inefficient to do lots of small jumps. Split 13 into parts (10 and 3). This gives you two jumps (10x7 and 3x7).
6x124= 124 books were sold. Each book cost £6. How much money was taken? | 100 20 4 6 | **600 120 24** = **744**	This is called the grid method. 124 is split into parts (100, 20 and 4) and each of these is multiplied by 6. The three answers are then added together.
72x34= A cat is 72cm long. A tiger is 34 times longer. How long is the tiger? | 70 2 30 | **2100 60** = **2160** 4 | **280 8** = **288** **2448** 1	This method also works for 'long multiplication'. Again split up the numbers and multiply each part. Add across the rows, then add those two answers together.

724x3= A TV costs £724. How much do 3 TVs cost? 7 2 4 x 3 2 1 7 2 2 1	When children are confident and understand the process of multiplication, they will then start to develop their understanding of the traditional short method. ***This is nationally expected by the end of year 4.***
72x34= A cat is 72cm long. A tiger is 34 times longer. How long is the tiger? 7 2 x 3 4 8 (4 x 2) 2 8 0 (4 x 70) 6 0 (30 x 2) + 2 1 0 0 (30 x 70) 2 4 4 8 1	Finally, children will move on to the long method of multiplication.
72x34= A cat is 72cm long. A tiger is 34 times longer. How long is the tiger? 7 2 x 3 4 2 8 8 2 1 6 0 2 4 4 8 1	Which some children manage to shorten into this version.

WRITTEN METHODS FOR DIVISION

6÷2= 6 Easter eggs are shared between 2 children. How many eggs do they get each? 	More pictures! Drawing often gives children a way into solving the problem.
12÷4= 4 apples are packed in a basket. How many baskets can you fill with 12 apples? ●●●● ●●●● ●●●●	Dots or tally marks can either be shared out one at a time or split up into groups. Encouraging children to use or visualise arrays of numbers or number lines can often help, depending on what the child prefers.
28÷7= A chew bar costs 7p. How many can I buy with 28p?  0 7 14 21 28	To work out how many 7's there are in 28, draw jumps of 7 along a number line. This shows you need 4 jumps of 7 to reach 28.
84÷6= I need 6 drawing pins to put up a picture. How many pictures can I put up with 84 pins?  10 groups 4 groups	It would take a long time to jump in sixes to 84 so children can jump on in bigger 'chunks'. A jump of 10 groups of 6 takes you to 60. Then you need another 4 groups of 6 to reach 84. Altogether, that is 14 sixes.
192÷8= 8 pencils fit in each packet. If you have 192 pencils, how many packets can be filled? 192 = **160** + **32** ↙ ↘ 20 groups + 4 groups = 24	It is helpful to split 192 into sensible 'chunks' before dividing. As you are dividing by 8, the 'chunks' chosen must also be multiples of 8. Divide each 'chunk' (how many groups of 8?) and then add the answers together.

184÷7= I need 184 chairs for a concert. I arrange them in rows of 7. How many rows do I need? 140 **20 groups** + 42 **6 groups** 182 2 more for the target number so… = 26 r2	There are two ways to 'chunk'. The easiest way, that develops earlier methods, is to 'chunk' up, using addition and/or multiplication facts. Here children are encouraged to remember easy multiplication facts: x10, x5, x2 and x1. They use these 'known facts' to set up a partial multiplication facts table and use these chunks to get close to the target number. This is easier, as it relates to the children's mental methods and their work with a number line.
184÷7= I need 184 chairs for a concert. I arrange them in rows of 7. How many rows do I need? 184 - 140 **20 groups** 44 - 42 **6 groups** 2 = 26 r2	Some children prefer to use 'chunks' to take away from the target number. In this example, you are taking away chunks of 7. First subtract 140 (20 groups of 7) and you are left with 44. Then subtract 42 (6 groups of 7), to leave 2. Altogether, that is 26 sevens with a remainder of 2.
026 r2 7)184	When children really understand what division is all about and they have sound mental methods, we encourage them to use the 'bus stop' method. ***This is nationally expected by the end of year 5.***
432 ÷ 15 becomes 28 r 12 15)432 300 132 120 12 Answer: 28 remainder 12	Finally children will be introduced to long division in Upper Key Stage 2 (Year 5 or 6 depending on progress).

LEARNING MULTIPLICATION FACTS

We all remember our teachers telling us how important it is to 'learn your tables'. Some children enjoy learning their multiplication facts by rote. However, we can build on a lot of the mental strategies above to make the task less onerous:

Connections: Chanting tables is only part of committing multiplication facts to memory. While the time tables are neat, they have the disadvantage of keeping calculations such as 6 x 7 and 7 x 6 separate. Often children who have learned up to the six times table do not realise they know much of the seven times table from the facts already covered. And while some pupils do enjoy chanting tables, many do not. Teaching the multiplication facts strategically helps children to make connections and reduces the burden on their memory.

Number sentences multiplication and division: Make sure that children are secure in the knowledge of triples - the sets of three numbers linked together by multiplication and division, for example 3, 8, 24. They should know that 3 x 8 = 24, 8 x 3 = 24, 24 ÷ 3 = 8, 24 ÷ 8 = 3. A simple teaching aid for this is a set of triangular cards with number from a triple placed in each of the corners. Cover up one of the numbers and invite children to express the multiplicative relationship between the remaining two. 3 multiplied by what is 24? What is 24 divided by 3? and so forth.

Commutativity: the order of the numbers doesn't affect the answer. For example, 3 x 4 is the same result as 4 x 3. This immediately reduces the number of multiplication facts to remember by almost half. An array model helps make this clear. Rotating the array by 90° shows that four rows of three is the same as three rows of four.

Encourage to use the small number first: When adding, we encourage children to start with the larger number. For example, if still at the stage of counting on when adding 4 + 17, it's easier to count on 4 from 17 than to count on 17 from 4. Children use skip-counting strategies when beginning with their tables and asked to calculate say, 7 x 3, will count 3, 6, 9, 12, 15, 18, 21. Drawing on the commutative property, we might take a lesson from Japanese pupils who are encouraged to put the smaller number first: instead of 7 x 3, calculate 3 x 7. Now you only have to count on 7, 14, 21. Nine times four? Don't do that, do four times nine!

Doubling and halving: A knowledge of doubles is central to committing multiplication facts to memory. Being able to double doesn't just give you the two times table - it also means you can quickly remind yourself what, say, 8 x 6 comes to if you've forgotten:

Double six: 2 x 6 = 12
Double again: 4 x 6 = 24
Double again: 8 x 6 = 48

Arrays are again helpful models for showing why this works.

Place value: Children need to be fluent in multiplying by 10 and later by 100. They might say that the shortcut method is to "add a zero". However, this rule does not hold for multiplying decimal numbers so it is better to point out that multiplying by 10 makes everything 10 times bigger. Base 10 blocks are useful here: setting out, say, 24 as two 10- sticks and four units, each of these becomes 10 times bigger. Each 10 stick needs to be replaced by a 100 square, and each unit replaced by a 10 stick: 24 gets scaled up by a factor of 10 to become 240. The result is that all the digits move one place to the left.

Compensation: A way of exploring the nines pattern is to multiply by 10 and subtract the number being multiplied. So nine times four is forty minus four, that's 36.

Arrays and shapes: The square numbers - 1 x 1, 2 x 2 up to 10 x 10 and beyond - are the cornerstones of the multiplication facts. If children know say, 6 x 6 = 36, they can easily work out 7 x 6 by adding on another 6 to get 42. Again, working with arrays and the pattern of how square numbers grow can help children commit these to memory.

What about the 7 and 8 times tables? Confident doublers will appreciate that multiplying by eight can be done by doubling, doubling and doubling again. If children know that multiplication is commutative, they can turn around most of the seven and eight times table facts - 7 x 5 becomes 5 x 7. Only three facts are then not covered in the other tables - 7 x 7, 8 x 8, and 7 x 8. The first two are covered by knowing the square numbers. Seven eights? Well, maybe you just have to remember that one - it's a table fact that everyone finds most difficult to recall! Although you might note that 5, 6, 7, 8 can turn into 56 = 7 x 8.

If children are confident with these strategies, they will become fluent with the multiplication and division facts up to 10 x 10. A bonus to this approach is that combining strategies allows them to work mentally beyond 10 x 10. For example, appreciating that doubling and doubling again is the same as multiplying by four makes 36 x 4 a reasonable mental calculation. Reversing this - halving and halving again - gives a useful strategy for mentally figuring out 76 divided by four. Combining doubling with multiplying by 10 makes mental multiplication by 20 possible. Figuring out 38 x 5 by halving 380 (38 x 10) is more efficient than multiplying the 30 and 8 x 5 separately.

For more information take a look at our times tables booklet!

IDEAS FOR HELPING YOUR CHILD

Real Life problems:

- ✶ Go shopping with your child to buy two or three items. Ask them to work out the total amount spent and how much change you will get.
- ✶ Buy some items with a percentage extra free. Help your child to calculate how much of the product is free.
- ✶ Plan an outing during the holidays. Ask your child to think about what time you will need to set off and how much money you will need to take.
- ✶ Use a TV guide. Ask your child to work out the length of their favourite programmes. Can they calculate how long they spend watching TV each day / each week?
- ✶ Use a bus or train timetable. Ask your child to work out how long a journey between two places should take? Go on the journey. Do you arrive earlier or later than expected? How much earlier/later?
- ✶ Help your child to scale a recipe up or down to feed the right amount of people.
- ✶ Work together to plan a party or meal on a budget.

Counting:

- ◎ Practise chanting for the number names. Encourage your child to join in with you. When they are confident, try starting from different numbers - 4, 5, 6 . . .
- ◎ Sing number rhymes together - there are lots of commercial tapes and CD's available.
- ◎ Give your child the opportunity to count a range of interesting objects (e.g. coins, pasta shapes or buttons). Encourage them to touch and move each object as they count.
- ◎ Count things you cannot touch or see (more difficult!!). Try lights on the ceiling, window panes, jumps, claps or oranges in a bag.

- Play games that involve counting (e.g. snakes and ladders, dice games, games that involve collecting objects).
- Look for numerals in the environment. You can spot numerals at home, in the street or when out shopping.
- Cut out numerals from newspapers, magazines or birthday cards. Then help your child to put the numbers in orders.
- Make mistakes when chanting, counting or ordering numbers. Can your child spot what you have done wrong?
- Choose a number of the week e.g. 5. Practise counting to 5 and on from 5. Count out groups of 5 objects (5 dolls, 5 bricks, 5 pens). See how many places you can spot the numeral 5.

Shape and space:

- Choose a shape of the week e.g. cylinder. Look for this shape in the environment (tins, candles etc). Ask your child to describe the shape (2 circular faces, 2 curved edges ..)
- Play 'guess my shape'. You think of a shape. Your child asks questions to try to identify it but you can only answer 'yes' or 'no' (e.g. Does it have more than 4 corners? Does it have any curved sides?)
- Hunt for right angles around your home. Can your child also spot angles bigger or smaller than a right angle?
- Look for symmetrical objects. Help your child to draw or paint symmetrical pictures / patterns?
- Make a model using boxes/containers of different shapes and sizes. Ask your child to describe their model.
- Practise measuring the lengths or heights of objects (in metres or cm). Help your child to use different rulers and tape measures correctly. Encourage them to estimate before measuring.
- Let your child help with cooking at home. Help them to measure ingredients accurately using weighing scales or measuring jugs. Talk about what each division on the scale stands for.

- Choose some food items out of the cupboard. Try to put the objects in order of weight, by feel alone. Check by looking at the amounts on the packets.
- Practise telling the time with your child. Use both digital and analogue clocks. Ask your child to be a 'timekeeper' (e.g. tell me when it is half past four because then we are going swimming).
- Use a stop clock to time how long it takes to do everyday tasks (e.g. how long does it take to get dressed?). Encourage your child to estimate first.

Number facts:

- Find out which number facts your child is learning at school (addition facts to 10, times tables, doubles etc). Try to practise for a few minutes each day using a range of vocabulary.
- Have a 'fact of the day'. Pin this fact up around the house. Practise reading it in a quiet, loud, squeaky voice. Ask your child over the day if they can recall the fact.
- Play 'ping pong' to practise complements with your child. You say a number. They reply with how much more is needed to make 10. You can also play this game with numbers totalling 20, 100 or 1000. Encourage your child to answer quickly, without counting or using fingers.
- Throw 2 dice. Ask your child to find the total of the numbers (+), the difference between them (-) or the product (x). Can they do this without counting?
- Use a set of playing cards (no pictures). Turn over two cards and ask your child to add or multiply the numbers. If they answer correctly, they keep the cards. How many cards can they collect in 2 minutes?
- Play Bingo. Each player chooses five answers (e.g. numbers to 10 to practise simple addition, multiples of 5 to practise the five times tables). Ask a question and if a player has the answer, they can cross it off. The winner is the first player to cross off all their answers.

- Give your child an answer. Ask them to write as many addition sentences as they can with this answer (e.g. 10 = □ + □). Try with multiplication or subtraction.
- Give your child a number fact (e.g. 5+3=8). Ask them what else they can find out from this fact (e.g. 3+5=8, 8-5=3, 8-3=5, 50+30=80, 500+300=800, 5+4=9, 15+3=18). Add to the list over the next few days. Try starting with a x fact as well.

GOOD WEBSITES

www.bbc.co.uk/schools/websites/4_11/site/numeracy.shtml
www.coolmath4kids.com/
http://nrich.maths.org/public/
http://www.woodlands-junior.kent.sch.uk/maths/index.html

Also, remember to check the school website (http://www.marlbrookschool.com/Maths).

If you would like more ideas about how to help your child, don't hesitate to contact me.

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MAKE MATHS FUN!

Give your child lots of help and encouragement!